Repairable Two Phase Service $M^x/G/1$ Queuing Models with Infinite Number of Immediate Feedbacks under Bernoulli Schedule Vacation and Repetition of Service during Breakdown

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ABSTRACT
In the present paper the author considers an unreliable $M^X/G, G_{i\ (1 \le i \le c)}/1$ queuing system with two phases of heterogeneous service in which the server operates single service in the first phase and multi-optional heterogeneous service facilities in second phase. The arriving customers have to undergo the first phase service and any one of the second optional services to complete the first round of service. After completing the first round of service, the customers may demand for re-services from the second phase, infinitely many times before leaving the system. The server is subject to unpredictable breakdowns during busy period and sent for repair immediately .During breakdown period, the service interrupted customers need to repeat the corresponding services. It is further assumed that, after a successful completion of a service (including the feedback services) of each customer the server may take a Bernoulli schedule vacation before starting a new service for the next customer.

KEYWORDS: *Batch Arrival, Bernoulli Schedule vacation, Infinite feedback, Multi-Optional services, Unreliable with repetition of service.*

I.INTRODUCTION

In feedback queuing models, if the service of a job of a customer is unsuccessful then, the customer tries the job again and again until a successful service is completed. Queuing systems with various feedback policies have been investigated by many authors. The concept of feedback was introduced by Takacs (1963) and since then many papers have appeared about this topic. He considered an M/G/1 Bernoulli feedback queue with single class customers and obtained the distributions of queue size and the total response time of a customer. Disney and Konig (1985) have given an overview of the literature concerning Bernoulli feedback studies. Fewer results are known for feedback queuing systems in which the feedback policy is not Bernoulli. Baskett et al. (1975) obtained the product form of the joint queue size distribution for the M/M/1 queuing system with several types of customers and general feedback policy. Thangaraj and Vanitha (2010), Choi and Tae-Sung (2003), Badamchi Zadeh and Shahkar (2008) and Choudhury and Paul (2005) derived the queue size distribution for M/G/1 queue with two phases of heterogeneous services and Bernoulli feedback system. Saravanarajan and Chandrasekaran (2014) analysed MX/G/1 feedback queue with two-phase service, compulsory server vacation and random breakdowns. Kalidass and Kasturi (2013) have considered a reliable Poisson arrival M/G/1 queuing system with two phases of heterogeneous services and finite number (m) of immediate Bernoulli feedbacks. Fijy and Afthab (2016) analyzed a repairable two phase

Service Mx/G/1 Queuing Models with infinite number of immediate feedbacks under Bernoulli Schedule Vacation in which the feedback customers can demand re-services either from the first phase or from the second phase and during breakdown period the customer will wait in the system to resume the service. In the present paper during breakdown period it is assumed that the service gets started from the very beginning of the service independently of the earlier amount of service and also the customer can repeat only the optional services of second stage during feedback services.

1.1 MATHEMATICAL ANALYSIS OF THE SYSTEM

1.1.1 Model Description

The present paper deals with $M^X/G/1$ queuing system with two-phases of service and Bernoulli vacation schedule for an unreliable server which consists of a breakdown period. The customers arrive at the system in batches of variable size in accordance with time-homogeneous compound Poisson process with group arrival rate λ.

Service is provided one by one according to FCFS basis. Every customer has to undergo two stages of services following different general (arbitrary) distributions. The arriving customers first receive the First Phase Service S (FPS), which is followed by any of the second phase services (SPS) S_i $(1 \le i \le C)$. The customers after completing the first phase service, can choose any of the i^{th} optional services (available in second phase) with probability r_i, where $\sum_{i=1}^{C} r_i = 1$. The second phase service commences immediately after the completion of first phase service, and all the services are provided by the same server. A customer is said to complete the first round of service if he undergoes the FPS and anyone of the i^{th} second optional services with probability r_i $(1 \le i \le C)$. The first round of service may be termed as primary (or) fresh service.

The customer, who finishes the first round of service r feeds back immediately from any of the ℓ^{th} second optional services with probability f r_ℓ $(1 \le \ell \le C)$ or leaves the system with probability $(1 – f)$. After the completion of the first feedback service, the customer may again go in for a third round of service and so on in a similar way. Thus the first round service consists of services of length $S + S_i$ $(1 \le i \le C)$. Second round of service (i.e., first feedback service) is S_ℓ with probability f r_ℓ (or) else does not exist with probability $(1 –f)$. This process will continue any number of times until the customer is satisfied. The next customer in the queue can go into the system only after the successful completion of all feedback rounds of the preceding customer. The distribution functions, density functions, LST of FPS and SPS are respectively denoted by $S(t)$, $S_i(t)$; $s(t)$, $s_i(t)$, $S^*(\theta)$, $S_i^{\cdot}(\theta)$ with finite moments.

During busy period, the server is subject to breakdowns. The lifetime of the server follows exponential distribution with parameters: a_1 in the FPS ; $a_2^{(i,0)}$ in the primary i^{th} second phase service and a_2^i in the i^{th} second optional feedback service.

The server whenever breaks down is sent for repair immediately and the customer just being served will repeat the service from the very beginning.

The repair time distributions of the server follow arbitrary distributions $R_1^0(t)$, $(R_2^{(i,0)})^o(t)$ and $(R_2^i)^o(t)$ respectively according as the breakdowns occur in first phase (or)second i^{th} primary or feedback services. It is also assumed that after completing a service to a customer (i.e., when the customer leaves the system) the server may take a Bernoulli Scheduled Single Vacation (BSV) with probability **p** or continue to serve the next customer in the queue if any (or) stay idle in the system for the next batch to arrive, with probability **(1 – p)**.

The vacation time V follows general distribution with its distribution function V(t), density function v(t), LST $V^*(\theta)$ with finite first and second moments $E(V^k)$, k = 1, 2.

Thus a cycle consists of primary services, feedback services, breakdown period and vacation period. Various stochastic processes involved in the queueing system are assumed to be independent of each other. The customers continue to arrive and join the system independent of the system states, following the compound Poisson process. Using supplementary variable technique the steady state system equations under the steady-state condition are analysed and the PGF of the system size is obtained so that various performance measures of the model can be derived from it.

The notations of the Random Variables (RV), Cumulative Distribution Function (CDF), Probability Density Function (PDF), Laplace-Stieltjes Transform (LST) and its kth moments of the R.Vs are similar to Section 1.1, Table 1.1.1. of Fijy and Afthab (2016).Thus the states of the system is Y(t) = 0, 1, 2, 3, 4, 5, 6 and 7 represents that the server is idle in the system, busy in first phase, busy in second service, in breakdown state during the first phase service, breakdown state during the second phase fresh service, breakdown state during the second feedback service and the server is in vacation respectively.

The joint probabilities corresponding to the breakdown states are explained below. The other system size probabilities are similar to 1.1 of Fijy and Afthab (2016)

$BR_{1,n}(y, t)\,dt$ = $\Pr\{N_S(t) = n,\ y < R_1^o(t) \Box y + dt,\ Y(t) = 3\},\ n \Box 1$, the service interrupted customer repeats a new first service and the repair time lies in the interval (y + y+dt).

$BR_{2,n}^{(i,0)}(y, t)\,dt$ = $\Pr\{N_S(t) = n,\ y < (R_2^{(i,0)})^o(t) \Box y + dt,\ Y(t) = 5\},\ n \Box 1,\ 1 \Box i \Box C$, the customer whose service is interrupted repeats a new second i^{th} primary service.

$BR_{2,n}^{i}(y, t)\,dt$ = $\Pr\{N_S(t) = n,\ y < (R_2^{i})^o(t) \Box y + dt,\ Y(t) = 6\},\ n \Box 1,\ 1 \Box i \Box C$, the customer whose service is interrupted repeats a new second i^{th} feedback service.

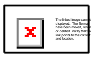
Using supplementary variable technique the steady state system equations under the steady-state condition are analysed and the PGF of the system size is obtained.The LST of the steady state equations are listed below :

$$\Box\ Q_n^*(\theta) \Box Q_n(0) = \Box\ Q_n^*(\theta) \Box \Box (1 - \Box_{0,n}) \sum_{k=1}^{n} Q_{n-k}^*(\theta)\, g_k \Box \sum_{i=1}^{C} (P_{2,n+1}^{(i,0)}(0) + P_{2,n+1}^{i}(0))(1 - f)\, p\ V^*(\theta), \quad n \Box 0 \tag{1.1}$$

$$\Box\ P_{1,n}^*(\theta) \Box P_{1,n}(0) = (\Box + a_1)\ P_{1,n}^*(\theta) \Box (1 - \Box_{1,n}) \Box \sum_{k=1}^{n-1} P_{1,n-k}^*(\theta)\, g_k \Box \sum_{i=1}^{C} (P_{2,n+1}^{(i,0)}(0) + P_{2,n+1}^{i}(0))\ (1 - f)(1 - p)\ S^*(\theta) \Box P\Box \Box g_n\ S^*(\theta) \Box BR_{1,n}(0)\ S^*(\theta) \Box Q_n(0)\ S^*(\theta), \quad n \Box 1 \tag{1.2}$$

$$\Box\ P_{2,n}^{(i,0)*}(\theta) \Box P_{2,n}^{(i,0)}(0) = (\Box + a_2^{(i,0)})\, P_{2,n}^{(i,0)*}(\theta) \Box (1 - \Box_{1,n}) \Box \sum_{k=1}^{n-1} P_{2,n-k}^{(i,0)*}(\theta)\, g_k \Box P_{1,n}(0)\, r_i\ S_i^*(\theta) \Box BR_{2,n}^{(i,0)}(0)\, S_i^*(\theta) \quad n \Box 1 \tag{1.3}$$

$$\Box\ P_{2,n}^{i*}(\theta) \Box P_{2,n}^{i}(0) = (\Box + a_2^{i})\, P_{2,n}^{i*}(\theta) \Box (1 - \Box_{1,n}) \Box \sum_{k=1}^{n-1} P_{2,n-k}^{i*}(\theta)\, g_k$$

$$\square \sum_{\ell=1}^{C} (P_{2,n}^{\ell}(0) + P_{2,n}^{(\ell,0)}(0))\, f\, r_i \;\; S_i^{*}(\theta) \;\square\; BR_{2,n}^{i}(0)$$ 

$n \square 1 \qquad (1.4)$

$$\square\; BR_{1,n}^{*}(\theta) \;\square\; BR_{1,n}(0) \;=\; \square\; BR_{1,n}^{*}(\theta) \;\square\; (1 \square \square_{1,n}) \;\square\; \sum_{k=1}^{n-1} BR_{1,n-k}^{*}(\theta)\, g_k$$

$$\square\; (\int_0^{\infty} P_{1,n}(w)\,dw)\, a_1 \;\; R_1^{*}(\theta), \qquad n \square 1 \qquad (1.5)$$

i.e., $\square\; BR_{1,n}^{*}(\theta) \;\square\; BR_{1,n}(0) \;=\; \square\; BR_{1,n}^{*}(\theta) \;\square\; (1 \square \square_{1,n}) \;\square\; \sum_{k=1}^{n-1} BR_{1,n-k}^{*}(\theta)\, g_k$

$$\square\; P_{1,n}^{*}(0)\, a_1 \;\; R_1^{*}(\theta), \qquad n \square 1 \qquad (1.5.1)$$

$$\square\; BR_{2,n}^{(i,0)^{*}}(\theta) \;\square\; BR_{2,n}^{(i,0)}(0) \;=\; \square\; BR_{2,n}^{(i,0)^{*}}(\theta) \;\square\; (1 \square \square_{1,n}) \;\square\; \sum_{k=1}^{n-1} BR_{2,n-k}^{(i,0)\;*}(\theta)\, g_k$$

$$\square\; (\int_0^{\infty} P_{2,n}^{(i,0)}(w)\,dw)\, a_2^{(i,0)}\; R_2^{(i,0)^{*}}(\theta), \; n \square 1 \qquad (1.6)$$

i.e., $\square\; BR_{2,n}^{(i,0)^{*}}(\theta) \;\square\; BR_{2,n}^{(i,0)}(0) \;=\; \square\; BR_{2,n}^{(i,0)^{*}}(\theta) \;\square\; (1 \square \square_{1,n}) \;\square\; \sum_{k=1}^{n-1} BR_{2,n-k}^{(i,0)\;*}(\theta)\, g_k$

$$\square\; P_2^{(i,0)^{*}}(0)\;\; a_2^{(i,0)}\; R_2^{(i,0)^{*}}(\theta), \qquad n \square 1 \qquad (1.6.1)$$

$$\square\; BR_{2,n}^{i*}(\theta) \;\square\; BR_{2,n}^{i}(0) \;=\; \square\; BR_{2,n}^{i*}(\theta) \;\square\; (1 \square \square_{1,n}) \;\square\; \sum_{k=1}^{n-1} BR_{2,n-k}^{i*}(\theta)\, g_k$$

$$\square\; (\int_0^{\infty} P_{2,n}^{i}(w)\,dw)\, a_2^{i}\; R_2^{i\;*}(\theta), \qquad n \square 1 \qquad (1.7)$$

$$\square\; BR_{2,n}^{i*}(\theta) \;\square\; BR_{2,n}^{i}(0) \;=\; \square\; BR_{2,n}^{i*}(\theta) \;\square\; (1 \square \square_{1,n}) \;\square\; \sum_{k=1}^{n-1} BR_{2,n-k}^{i*}(\theta)\, g_k$$

$$\square\; P_2^{i\;*}(0)\, a_2^{i}\; R_2^{i\;*}(\theta), \qquad n \square 1 \qquad (1.7.1)$$

$$\square\; P\square \;= Q_0(0) + \sum_{i=1}^{C} (P_{2,1}^{(i,0)}(0) + P_{2,1}^{i}(0))\,(1-f)\,(1-p)$$

(1.8)

After some algebraic manipulations (Fijy and Afthab (2016)) the partial generating functions corresponding to different states at arbitrary epochs are calculated using the respective equations and are given by

$$P_1^{*}(z,0) \;=\; \frac{PI\, z\,(S^{*}(g_{a_1}(w_X(z))) - 1)\, w_X(z)}{D_{2,if}^{BV}(z)} \qquad (1.9)$$

where $D_{2,if}^{BV}(z)$

$$= \; z\,[h_{a_1}(w_X(z)) \;+\; S^{*}(g_{a_1}(w_X(z)))\, a_1 \;\; R_1^{*}(w_X(z))]$$

$$\square\, S^{*}(g_{a_1}(w_X(z)))\; g_{a_1}(w_X(z))\;\; [(1-f)\,(1-p+p\;\; V^{*}(w_X(z)))\, \frac{K_0(z)}{1 - f\,K(z)}]$$

$$g_{a_1}(w_X(z)) \;=\; a_1 + w_X(z)\,;$$

$$h_{a_1}(w_X(z)) \;=\; g_{a_1}(w_X(z)) \;\square\; a_1 \;\; R_1^{*}(w_X(z))$$

For $1 \square i \square C$

$$g_{a_2^i}(w_X(z)) = a_2^i + w_X(z)$$

$$h_{a_2^i}(w_X(z)) = g_{a_2^i}(w_X(z)) \Box a_2^i R_2^*(w_X(z)),$$

$$g_{a_2^{(i,0)}}(w_X(z)) = a_2^{(i,0)} + w_X(z)$$

$$h_{a_2^{(i,0)}}(w_X(z)) = g_{a_2^{(i,0)}}(w_X(z)) \Box a_2^{(i,0)} R_2^{(i,0)^*}(w_X(z))$$

$$K_0(z) = \sum_{i=1}^{C} \frac{r_i S_i^*(g_{a_2^{(i,0)}}(w_X(z)))\, g_{a_2^{(i,0)}}(w_X(z))}{h_{a_2^{(i,0)}}(w_X(z)) + S_i^*(g_{a_2^{(i,0)}}(w_X(z)))\, a_2^{(i,0)} R_2^{(i,0)^*}(w_X(z))}$$

$$K(z) = \sum_{i=1}^{C} \frac{r_i S_i^*(g_{a_2^i}(w_X(z)))\, g_{a_2^i}(w_X(z))}{h_{a_2^i}(w_X(z)) + S_i^*(g_{a_2^i}(w_X(z)))\, a_2^i R_2^{i^*}(w_X(z))}$$

$$P_2^{(i,0)^*}(z,0) = \frac{PI\, z\, r_i\, (S_i^*(g_{a_2^{(i,0)}}(w_X(z))) - 1)\, S^*(g_{a_1}(w_X(z)))\, g_{a_1}(w_X(z))}{D_{2,if}^{BV}(z)\ [h_{a_2^{(i,0)}}(w_X(z)) + S_i^*(g_{a_2^{(i,0)}}(w_X(z)))\, a_2^{(i,0)} R_2^{(i,0)^*}(w_X(z))]} \qquad (1.10)$$

$$P_2^{i^*}(z,0) = \frac{PI\, z\, r_i\, (S_i^*(g_{a_2^i}(w_X(z))) - 1)\, S^*(g_{a_1}(w_X(z)))\, g_{a_1}(w_X(z))\, w_X(z)}{(h_{a_2^i}(w_X(z)) + S_i^*(g_{a_2^i}(w_X(z)))\, a_2^i R_2^{i^*}(w_X(z)))\, D_{2,if}^{BV}(z)} \left(\frac{f K_0(z)}{1 - f K(z)}\right) \qquad (1.11)$$

$$Q^*(z,0) = \frac{PI\, p\ (1-f)\,(V^*(w_X(z)) - 1)\, S^*(g_{a_1}(w_X(z)))\, g_{a_1}(w_X(z))}{D_{2,if}^{BV}(z)} \left(\frac{K_0(z)}{1 - f K(z)}\right) \qquad (1.12)$$

$$BR_1^*(z,0) = \frac{a_1 P_1^*(z,0)(1 - R_1^*(w_X(z)))}{w_X(z)} \qquad (1.13)$$

$$BR_2^{(i,0)^*}(z,0) = \frac{a_2^{(i,0)} P_2^{(i,0)^*}(z,0)(1 - R_2^{(i,0)^*}(w_X(z)))}{w_X(z)} \qquad (1.14)$$

$$BR_2^{i^*}(z,0) = \frac{a_2^i P_2^{i^*}(z,0)(1 - R_2^{i^*}(w_X(z)))}{w_X(z)} \qquad (1.15)$$

The total PGF of the system size distribution is obtained by adding equations (1.9) to (1.15) and given by

$$P_{if}^{BV}(z) = \frac{PI\, g_{a_1}(w_X(z))\, S^*(g_{a_1}(w_X(z)))\,(z-1)(1-f)\, K_0(z)}{D_{2,if}^{BV}(z)\,(1 - f K(z))} \qquad (1.16)$$

The constant P□ can be calculated by using the normalizing condition $P_{if}^{BV}(1) = 1$ and found to be P□ $= 1 \Box \rho_{2,if}^{BV}$
where

$$\rho_{2,if}^{BV} = \square\, E(X)\ [\frac{(1-S_1^*(a_1))}{a_1\, S_1^*(a_1)}(1 + a_1\, E(R_1)) + p\, E(V)$$
$$+ \sum_{i=1}^{C} \frac{r_i\,(1+a_2^{(i,0)}\, E(R_2^{(i,0)}))\,(1-S_i^*(a_2^{(i,0)}))}{S_i^*(a_2^{(i,0)})\, a_2^{(i,0)}}$$
$$+ \sum_{i=1}^{C} [(\frac{f}{1-f})\frac{r_i\,(1+a_2^{i}\, E(R_2^{i}))\,(1-S_i^*(a_2^{i}))}{a_2^{i}\, S_i^*(a_2^{i})}]] \qquad (1.17)$$

Hence

$$P_{if}^{BV}(z) = \frac{(1-\rho_{2,if}^{BV})\, g_{a_1}(w_X(z))\, S^*(g_{a_1}(w_X(z)))\,(z-1)\,(1-f)\, K_0(z)}{D_{2,if}^{BV}(z)\,(1-f\,K(z))} \qquad (1.18)$$

1.2 PARTICULAR CASES

If the service times S and S_i (1 □ i □ C) follow exponential distribution with parameters □ and $\square_i$ respectively, then the steady state results of the model in which the service interrupted customers resume services coincide with that of the model in which the customers repeat service from the beginning as soon as the server is fixed. The following observations confirm the result:

Let $S^*(\theta) = \frac{\mu}{\mu+\theta}$ and $S_i^*(\theta) = \frac{\mu_i}{\mu_i+\theta}$. (1 □ i □ C)

This implies,

$$K(z) = k(z) = \sum_{i=1}^{C} r_i \frac{\mu_i}{\mu_i + h_{a_2^i}(w_X(z))} \quad \text{and} \quad K_0(z) = k_0(z)$$

Also

$$\rho_{2,if}^{BV} = \rho_{1,IF}^{BV} = \square\, E(X)\,\{\frac{f}{1-f}\sum_{i=1}^{C} r_i \frac{1}{\mu_i}(1+a_2^{i}\,E(R_2^{i})) + \sum_{i=1}^{C} r_i \frac{1}{\mu_i}(1+\ a_2^{(i,0)}\, E(R_2^{(i,0)})$$
$$+ \frac{1}{\mu}(1+\ a_1^{0}\, E(R_1^{0})) + p\, E(V)\}$$

By calculation, it is found that,

$$\frac{g_{a_1}(w_X(z))\, S^*(g_{a_1}(w_X(z)))}{D_{2,if}^{BV}(z)\,(1-f\,K(z))} = \frac{S^*(h_{a_1}(w_X(z)))}{D_{1IF}^{BV}(z)} = \frac{\mu}{(\mu + h_{a_1}(w_X(z)))\ D_{1IF}^{BV}(z)}$$

Thus the total PGFs given in equations (1.38 of Fijy and Afthab(2016)) and (1.18) coincide.

II Conclusion

The present paper examines $M^X/G/1$ queueing system with immediate feedbacks and multi second optional service facilities. The server operates single service in the first phase and different kinds of heterogeneous services in the second phase. A customer is said to complete the first round service if he undergoes the first phase service and any one of the second phase services. After having completed the first round service, the customer is permitted to repeat services from the second multi-optional services which may be different from the one chosen earlier. Fijy and Afthab(2016) studied the $M^X/G/1$ queueing system in which the feedback customers can demand re-services either from the first phase or from the second phase and when the server fails, the customer in service may resume the service. In the present paper during breakdown period it is assumed that the service gets started from the very beginning of the service independently of the earlier amount of service and also the customer can repeat only the optional services of second stage during feedback services.

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